[Technical Data]
Spring Calculations
Excerpts from JIS B 2704(2000)

1. Calculation

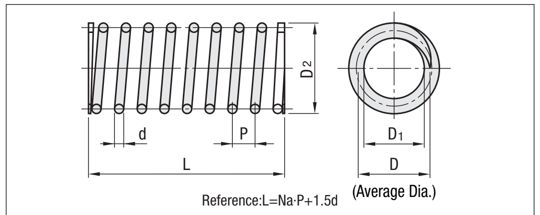
1.1 Symbols Used in Spring Design Formulae

Symbols used in spring design formulae are shown in Table 1.

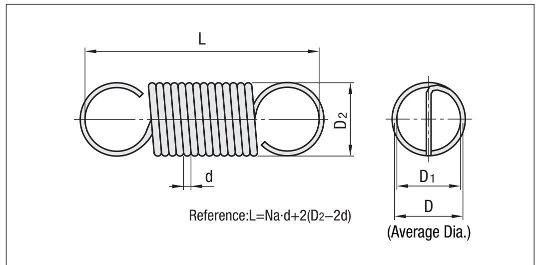
Table 1 Meaning of Symbols

Symbol	Meaning of Symbols	Unit
d	Diameter of Material	mm
D_1	Inner Diameter of a Coil	mm
D_2	Outer Diameter of a Coil	mm
D	Coil Mean Diameter= $\frac{D_1+D_2}{2}$	mm
N_t	Total Number of Winding	–
N_a	Number of Active Winding	–
L	Free Length(Length)	mm
Hs	Solid Length	mm
p	Pitch	mm
P_i	Initial Tension	N{kgf}
c	Spring Index $c=\frac{D}{d}$	–
G	Shear Modulus of Elasticity	$N/mm^2\{kgf/mm^2\}$
P	Load on Spring	N{kgf}
δ	Spring Deflection	mm
k	Spring Constant	N/mm{kgf/mm}
τ_0	Torsional Stress	$N/mm^2\{kgf/mm^2\}$
τ	Corrected Torsional Stress	$N/mm^2\{kgf/mm^2\}$
τ_i	Initial Stress	$N/mm^2\{kgf/mm^2\}$
χ	Stress Correction Factor	–
f	Frequency	Hz
U	Spring-Retained Energy	N·mm{kgf·mm}
ω	Per Unit Volume Material Weight	kg/mm^3
W	Mass of Moving Parts	kg
g	Gravitational Acceleration (1)	mm/s^2

Note (1) In spring calculations, a gravitational acceleration of 9806.65mm/s², is used.



Reference:L=Na·P+1.5d (Average Dia.)



Reference:L=Na·d+2(D2−2d) (Average Dia.)

1.2 Basic Formulae Used in Designing of Springs

1.2.1Compression Springs, and Tension Springs without Initial Tension

$$\delta=\frac{8N_aD^3P}{Gd^4} \quad \cdots (1)$$

$$k=\frac{P}{\delta}=\frac{Gd^4}{8N_aD^3} \quad \cdots (2)$$

$$\tau_0=\frac{8DP}{\pi d^3} \quad \cdots (3)$$

$$\tau_0=\frac{Gd\delta}{\pi N_aD^2} \quad \cdots (4)$$

$$\tau=\chi\tau_0 \quad \cdots (5)$$

$$d=\sqrt[3]{\frac{8DP}{\pi\tau_0}}=\sqrt[3]{\frac{8\chi DP}{\pi\tau}} \quad \cdots (6)$$

$$N_a=\frac{Gd^4\delta}{8D^3P}=\frac{Gd^4}{8D^3k} \quad \cdots (7)$$

$$U=\frac{P\delta}{2}=\frac{k\delta^2}{2} \quad \cdots (8)$$

1.2.2 Tension Springs with Initial Tension (Where:P>Pi)

$$\delta=\frac{8N_aD^3(P-P_i)}{Gd^4} \quad \cdots (1')$$

$$k=\frac{P-P_i}{\delta}=\frac{Gd^4}{8N_aD^3} \quad \cdots (2')$$

$$\tau_0=\frac{8DP}{\pi d^3} \quad \cdots (3')$$

$$\tau_0=\frac{Gd\delta}{\pi N_aD^2}+\tau_i \quad \cdots (4')$$

$$\tau=\chi\tau_0 \quad \cdots (5')$$

$$d=\sqrt[3]{\frac{8DP}{\pi\tau_0}}=\sqrt[3]{\frac{8\chi DP}{\pi\tau}} \quad \cdots (6')$$

$$N_a=\frac{Gd^4}{8D^3k}=\frac{Gd^4\delta}{8D^3(P-P_i)} \quad \cdots (7')$$

$$U=\frac{(P+P_i)\delta}{2} \quad \cdots (8')$$

1.3 Points to Note when Designing Springs

1.3.1 Shear Modulus of Elasticity

Shear modulus of elasticity(G)listed in Table 2 is recommended for the designing of springs.

Table 2 Shear Modulus of Elasticity(G)

Material		G Value $N/mm^2(kgf/mm^2)$	Symbol
Spring Steel		$78\times10^3\{8\times10^3\}$	SUP6,7,9,9A,10, 11A,12,13
Hard Steel Wire		$78\times10^3\{8\times10^3\}$	SW-B,SW-C
Piano Wire		$78\times10^3\{8\times10^3\}$	SWP
Oil Tempered Steel Wire		$78\times10^3\{8\times10^3\}$	SWO,SWO-V,SWOC-V, SWOSC-V,SWOSM,SWOSC-B
Stainless Steel Wire	SUS 302	$69\times10^3\{7\times10^3\}$	SUS 302
	SUS 304		SUS 304
	SUS 304N1		SUS 304N1
	SUS 316		SUS 316
	SUS 631 J1	$74\times10^3\{7.5\times10^3\}$	SUS 631 J1

1.3.2 Number of Active Winding

The number of active winding can be determined as follows.

(1) Compression Springs

$N_a=N_t-(X_1+X_2)$

Where X1 and X2: are the number of turns at each end of the coil.

(a) When only the end of the coil is in contact with the next free coil [Corresponding to (a) ~ (c) in Fig.2]

$X_1=X_2=1$

Therefore, $N_a=N_t-2$

(b) When the end of the coil is not in contact with the next coil, and the spring end has $\frac{3}{4}$ of a turn. [Corresponding to (a) ~ (e) in Fig.2]

$X_1=X_2=0.75$

Therefore, $N_a=N_t-1.5$

(2) Tension Springs The number of active winding can be determined as follows. But hooks are ignored.

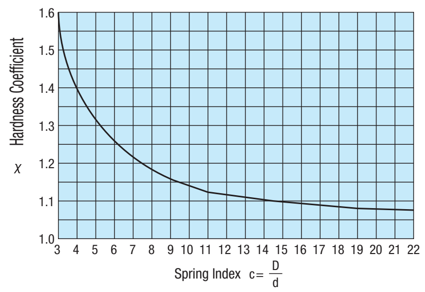
$N_a=N_t$

1.3.3 Stress Correction Factor

The stress correction factor relative to the spring index (C) can be determined by using the following formula or based on Fig.1.

$$\chi=\frac{4c-1}{4c-4}+\frac{0.615}{c} \quad \cdots (9)$$

Fig-1.Hardness Coefficient : χ



1.3.4 Solid Length

The solid length of a spring can normally be obtained by using the following simplified formula. Generally, the purchaser of a compression spring does not specify the solid length of the spring.

$$H_s=(N_t-1)d+(t_1+t_2) \quad \cdots (10)$$

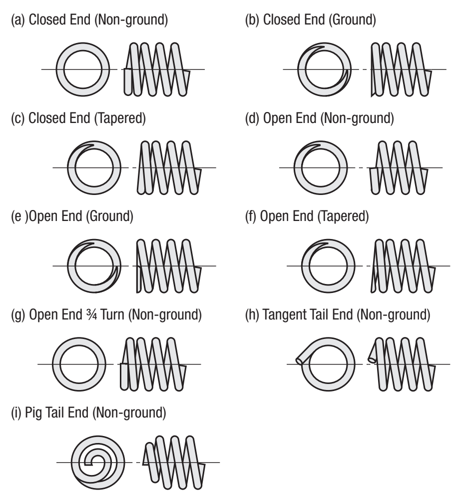
where, (t_1+t_2) : is the sum of the thicknesses of the coil ends.

As for those compression springs, both ends of which are shaped as shown in (b), (c), (e) or (f) of Figure 2 and for which the solid length needs to be specified, the following formula can be used to obtain the maximum solid length. However, the actual maximum solid length can be greater than the value thus calculated depending on the shape of the spring in question.

$$H_s=N_t\times d_{max} \quad \cdots (11)$$

where dmax : d is the material diameter with the maximum tolerance.

Fig-2. Coil End Shape



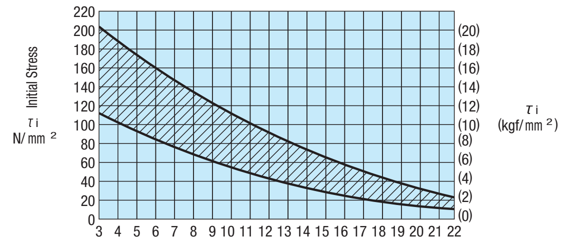
1.3.5 Initial Tension of Tension Springs

Cold-formed solid-coiled tension springs are subjected to initial tension (Pi)
The initial tension can be obtained using the following formula.

$$P_i=\frac{\pi d^3}{8D}\tau_i \quad \cdots (12)$$

On solid-coiled piano wire, hard steel wire, and other steel wires that are not low-temperature annealed, the initial stress occurs within the hatched range shown in Fig.3. However, if materials other than steel wire are used, or the wire in question is low-temperature annealed, the initial stress taken from within the hatched range in Fig.3 should be corrected as follows.

Fig-3 Initial Stress : τ_i (Spring formed from steel coil, not low-temperature annealed)



Spring Index $c=\frac{D}{d}$

(1) When using stainless steel wire, decrease the initial stress value for steel wire by 15%.

(2) If the spring is low-temperature annealed after being formed, decrease the value by 20~35% for springs made of piano wire, hard steel wire, or other stainless steel wires, and by 15~25% for springs made of stainless steel wire.

Reference In place of Fig.3, the following empirical formula can be used to establish the initial stress for springs before low-temperature annealing.

$$\tau_i=\frac{G}{100c}$$

The following examples are for applications of this formula to obtain the initial tension.

(1) Piano Wire / Hard Steel Wire $[G=78\times10^3N/mm^2\{8\times10^3kgf/mm^2\}]$

Initial Stress $\tau_i=\frac{G}{100c}\times0.75$ (0.75 by 25, reduction by low-temperature annealing).

Initial Tension $P_i=\frac{\pi d^3}{8D}\ \tau_i=\frac{Gd^4}{255D^2}\times0.75=\frac{229d^4}{D^2}\left\{\frac{24d^4}{D^2}\right\}$

(2) When using stainless steel wire $[G=69\times10^3N/mm^2\{7\times10^3kgf/mm^2\}]$

Initial Stress $\tau_i=\frac{G}{100c}\times0.8$(0.8 by 20, reduction by low-temperature annealing).

Initial Tension $P_i=\frac{\pi d^3}{8D}\ \tau_i=\frac{Gd^4}{255D^2}\times0.8=\frac{216d^4}{D^2}\left\{\frac{22d^4}{D^2}\right\}$

1.3.6 Surging

In order to prevent surging, the spring selected should be as that its natural frequency does not resonate with any of the natural frequencies that may act upon the spring.
The initial tension can be obtained using the following formula.

$$f=a\sqrt{\frac{kg}{W}}=a\frac{70d}{\pi N_aD^2}\sqrt{\frac{G}{\omega}} \quad \cdots (13)$$

Where, $a=\frac{i}{2}$:when both spring ends are either free or fixed

$a=\frac{2i-1}{4}$:When one spring end is fixed while the other end is free $i=1,2,3\cdots$

$G=78\times10^3N/mm^2\{8\times10^3kgf/mm^2\}$,
$w=76.93\times10^{-6}\ N/mm^3\{7.85\times10^{-6}kgf/mm^3\}$ If both spring ends are either free or fixed, the natural primary frequency of a spring can be obtained as follows.

$$f_1=3.56\times10^5\frac{d}{N_aD^2} \quad \cdots (13')$$

1.3.7 Other Points to Note

In spring design calculations, the following points should also be taken into account.

(1) Spring Index Excessive local stress can result from too small spring index. Machinability is compromised if the spring index is too great or small. The spring index should be selected from the range of 4~15 when hot forming, and from the range of 4~22 when cold forming.

(2) Slenderness Ratio In order to ensure the correct number of active winding, the slenderness ratio for a compression spring (Ratio of free height to coil mean diameter) should be 0.8 or greater. Furthermore, buckling considered, it is generally recommended that the slenderness ratio be selected from the range of 0.8 ~ 4 to prevent buckling.

(3) Number of Active Winding The number of active winding should be 3 or more in order to stabilize spring characteristics.

(4) Pitch Generally, when the pitch exceeds 0.5D, the spring deflection (load) increases to the extent that the coil diameter changes. This requires correction of the deflection and torsional stress values obtained by the basic formulae. Therefore, the pitch should be 0.5D or smaller. The pitch can generally be estimated using the following simplified formula.

$$p=\frac{L-H_s}{N_a}+d \quad \cdots (14)$$